Advanced Computational Methods to Understand & Mitigate Extreme Events

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Extreme events are situations where the loading and/or response of a material, structure, or system exceeds that of normal conditions. They can be naturally occurring or man-made, and they cross essentially all disciplines of science and engineering. These problems often define the research boundaries for science and technology in the sense of our ability to predict, analyze, and mitigate their effects. Examples include earthquakes, landslides, explosions *(Figure 1)*, projectile penetration, and hypervelocity space impact, to name a few.

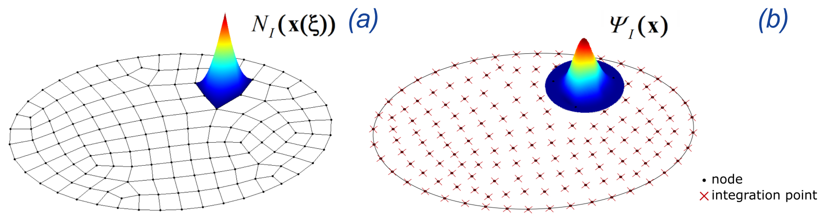
Figure 1: *Large-scale explosion, an example of an extreme event (courtesy of U.S. Army ERDC)*

In the field of extreme events research, physical experimentation is often limited according to the high rates, short time scales, large deformation response and/or physical size that are inherently present in these problems. Computation extends our capabilities beyond these experimental limitations, however methods suitable for modeling these events must be versatile and stable when dealing with:

- rough solutions in the form of transient strong discontinuities (fractures) and weak discontinuities (localization)
- severe material deformation and material instability
- multi-body contact without a priori knowledge of potential contacting bodies and contact surfaces
- multiple, evolving, and mixed length scales
- multiple and coupled physics

Meshfree methods such as the reproducing kernel particle method (RKPM) [1,2] offer unique features that are particularly attractive for modeling extreme events. By employing these methods, the domain can be discretized by a set of scattered nodes, and the approximation is constructed directly in the Cartesian coordinates with only nodal information. As seen in *Figure 2*, rather than a structured mesh defining connectivity as in the conventional finite element method, the interaction between points is achieved through the nodes' kernel coverage of any point in the domain. Consequently, issues related to mesh distortion, entanglement, alignment, and so on, are greatly relieved or completely circumvented. Adaptive refinement is also made easier as the conforming condition is relaxed to the partition of unity subordinate to open covering of the domain. The continuity of the approximation is entirely independent of the order of basis functions, allowing arbitrary smoothness (and roughness) in the approximation to be varied in space and time according to the physics of the problem at hand. Essential features such as crack-tip singularities and strong discontinuities can also be embedded in the approximation through intrinsic enrichment. The resulting purely node-based discretization is suitable for problems with damage, material flow, complex evolving multi-body contact, and fragmentation [3].

Figure 2: *(a) Construction of finite element shape functions by patching local element domains;*
(b) Reproducing kernel shape functions constructed directly in the Cartesian coordinates with direct nodal integration scheme



Weak-form based meshfree methods necessitate numerical quadrature. Nodal integration, as shown in *Figure 2(b)*, is a natural choice for this class of problems, as it is particularly advantageous for modeling material failure, fracture and separation that typifies extreme events. However, nodal integration constitutes low order quadrature and can yield poor accuracy. As seen in *Figure 3(a)*, the solution to PDEs by direct nodal integration (DNI) does not converge with refinement of non-uniform discretizations. Based on the framework of variational consistency [4], the variationally consistent integration (VCI) method has been introduced as a correction of any given quadrature to restore accuracy and optimal convergence. The method can be used to correct nodal integration methods such as DNI, stabilized non-conforming nodal integration (SNNI) [3] (the non-conforming version of stabilized conforming nodal integration [5]), and 2nd order Gauss quadrature (GI-2) to yield convergent solutions, as seen in *Figure 3(a)*.

While most natural for modeling material damage under extreme conditions, nodal integration is also subject to instability due to the severe underestimation of the strain energy of short-wavelength modes. As shown in *Figure 3(b)*, strains vanish at nodal locations for modes with a wavelength of two times the nodal spacing. As a consequence, they can grow virtually unbounded and destroy the numerical solution as seen in *Figure 3(c)* where Poisson's equation is solved using DNI for illustration. Naturally stabilized nodal integration (NSNI) [6] has recently been introduced to circumvent this difficulty, and also address other short-comings in nodal integration for modeling extreme events. The method yields stable solutions as shown in *Figure 3(d)*, where Poisson's equation is solved using NSNI. The method is based on the Taylor expansion-type stabilization that originated in finite elements [7], but employs implicit gradients [8] rather than direct derivatives in the expansion. NSNI can be employed in conjunction with VCI to yield a method that is stable, convergent, and efficient [6]. The method is free of conforming cells and background meshes, and is thus suitable for modeling extremely large deformations encountered in extreme events.

Many extreme events involve complex failure mechanisms across multiple length scales. A micro-crack informed damage model for describing the softening behavior of brittle solids has been proposed [9], in which the damage accumulation is treated as a consequence of micro-crack evolution.

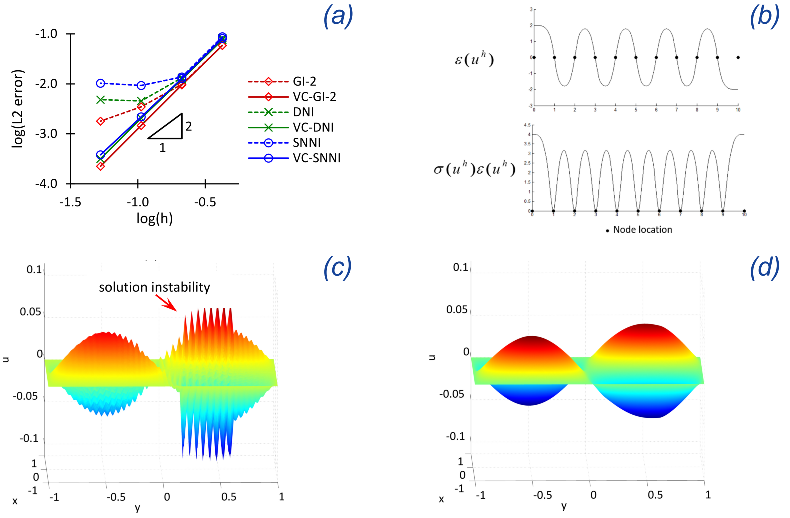
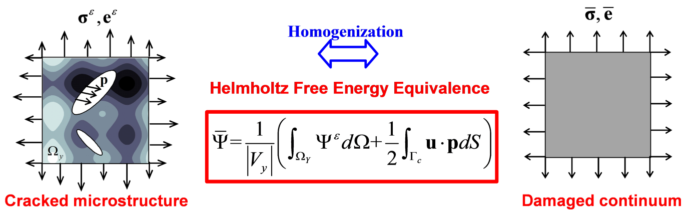


Figure 3:
Convergence of several low order quadrature schemes in a non-uniform discretization of $\Delta u + \sin(\pi x)\sin(\pi y) = 0$ on Ω, Ω: (-1,1) x (-1,1), $u = 0$ on $\partial\Omega$; "GI-2" 2x2 Gauss integration, "DNI" direct nodal integration, "SNNI" stabilized non-conforming nodal integration, prefix "VC-" variationally consistent; (b) Examination of strain energy associated with an oscillating displacement mode $u^h(x)=\sum_{L=1}^{NP}\Psi_L(x)u_L$, $u_L=(-1)^L$; Solution to $\Delta u + \sin(\pi x)\sin(\pi y) = 0$ on Ω, Ω: (-1,1) x (-1,1), $u = 0$ on $\partial\Omega$: (c) Solution by direct nodal integration, and (d) Solution by NSNI

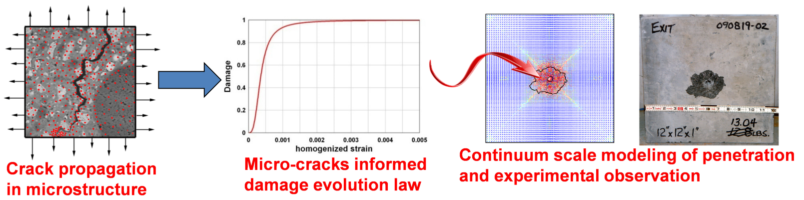
The Helmholtz free energy in the cracked microstructures is made equivalent to that of the damaged continuum as shown in *Figure 4(a)*, such that the micro-scale fracture models and the continuum-scale damage models exhibit equivalent energy density dissipation. This approach has been applied to fragment-impact modeling of concrete structures [10], and is shown in *Figure 4(b)*.

Figure 4.
(a) Micro-crack informed damage model [9]; (b) multi-scale modeling of material damage [10]

(a)



(b)



To illustrate the effectiveness of these methods, a suite of penetration problems from the blind prediction study in [11] is examined with SNNI [3] (a nodal integration) and NSNI [6] (a stabilized nodal integration). The problems

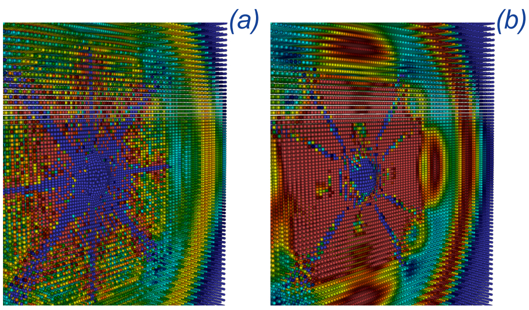


Figure 5:
Von Mises stress in concrete penetration: (a) SNNI and (b) NSNI.

Figure 6: *Cross-section of a penetration process using (a) SNNI, and (b) NSNI*

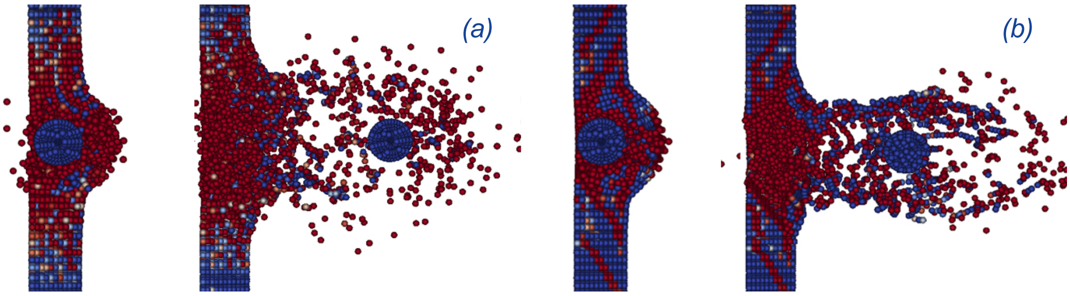
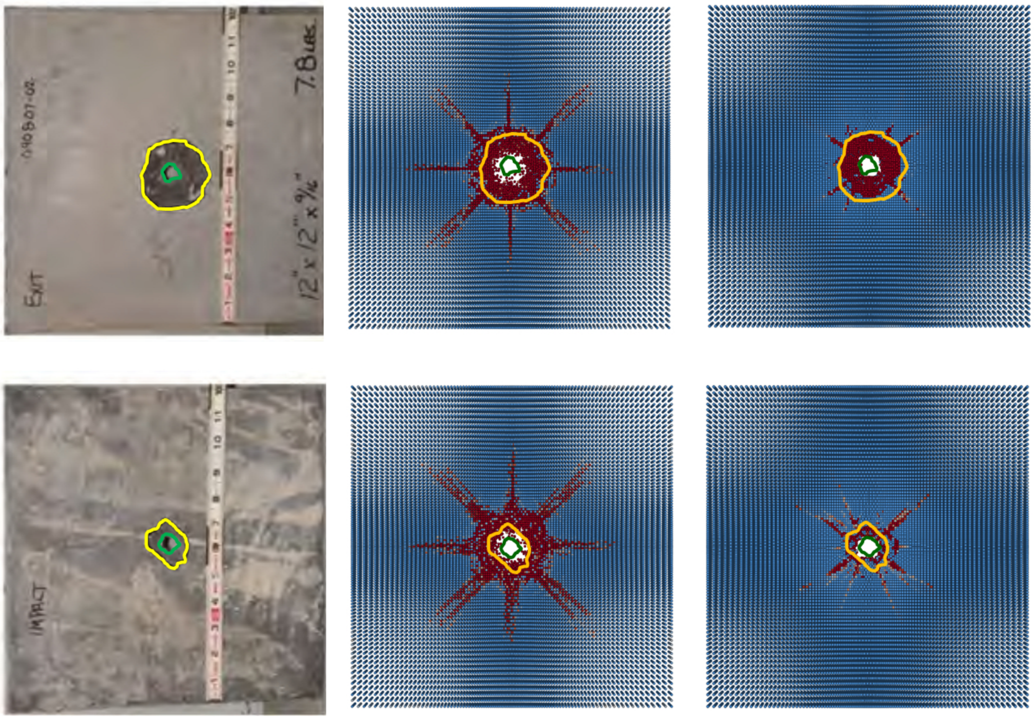
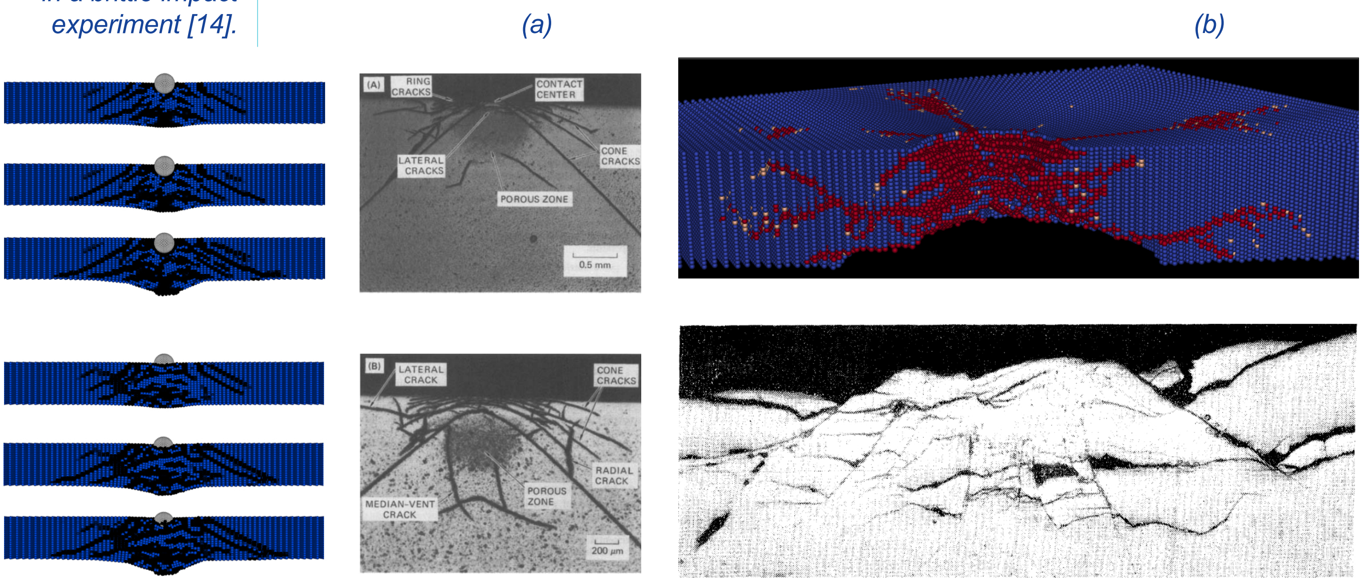


Figure 7: *Top to bottom: exit face damage, impact face damage. Left to right: experiment, SNNI, and NSNI*



in this study are representative of one of the many difficult classes of problems in the field of extreme events. Penetration of CorTuf concrete [12] panels with varying thicknesses and impact velocities is simulated. *Figure 5(a)* shows the von Mises stress from an SNNI simulation of the penetration process; checker-boarding, a multi-dimensional version of one-dimensional oscillating modes, is clearly observed in the stress field when pure nodal integration is employed. This results in unreasonable debris cloud shapes, and excessive, diffuse damage, as seen in

Figure 8: *(a) Numerical by NSNI and experimental [13] cracking patterns; Top: cross sections of the cracking pattern at the point of impact, Bottom: cross sections of the cracking pattern away from impact; (b) Top: lateral cracking in numerical simulations, Bottom: lateral cracking in a brittle impact experiment [14].*



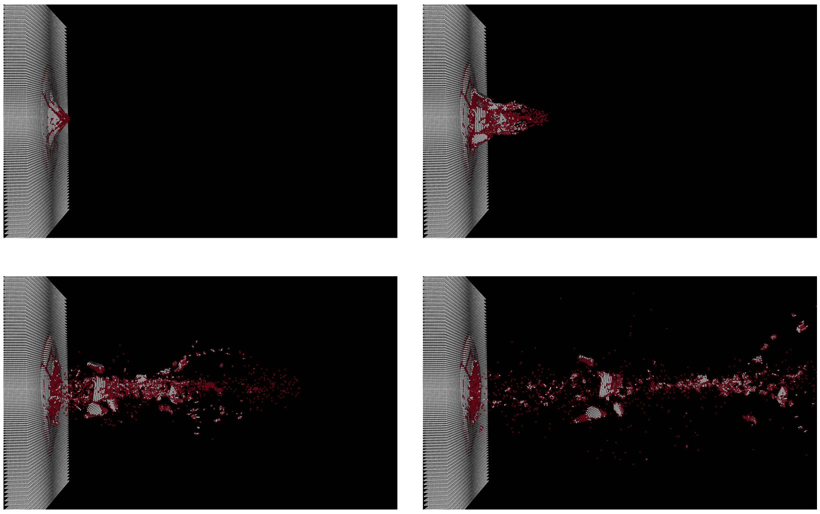


Figure 9:
Fragmentation of concrete target in a perforation event with large velocity reduction of the penetrator

Figure 6(a).
As a consequence, the expected shear-cone formation is almost completely absent for the SNNI solution, and the predicted crater and hole sizes are considerably larger than the experimental results *(Figure 7)*. In *Figure 5(b)* it is seen that the stability of NSNI avoids spurious checker-boarding, and captures clear radial cracking. Subsequently, shear-cone formation is captured when NSNI is employed as shown in *Figure 6(b)*. The NSNI results also show much better agreement with the experimental failure patterns in *Figure 7*.

The radial and lateral cracking patterns obtained by NSNI are in agreement to those experimentally observed in brittle material failure, as shown in *Figure 8*. The change in the dominant failure mode as a function of penetration conditions is observed in NSNI simulations as shown in *Figure 9* and *Figure 10*. For the case in *Figure 9* where there is a large velocity reduction of the penetrator i.e., the penetrator is nearly stopped by the target, and large pieces of intact debris are formed as the penetrator perforates the target, which is consistent with experimental observations. For cases where the penetrator overmatches the target and maintains significant exit velocity, the excessive kinematic energy is dissipated via high density material damage leading to debris cloud formation; qualitative agreement between experimental results and NSNI simulations is shown in *Figure 10*.

This computational framework can also be used to model impact and blast loads on ductile materials, where the ductile failure modes are properly captured. *Figure 11* shows a steel plate subject to a blast load from the UC San Diego blast simulator [16]. It is seen that the formulation captures the large ductile deformation and tearing at the edge

Figure 10:
Comparison of debris cloud in perforation of concrete target with moderate reduction in penetrator velocity:
(a) Numerical, *(b) Experimental [15]*

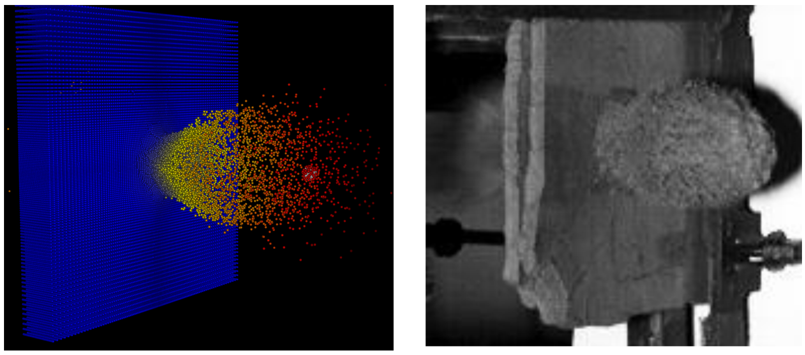


Figure 11:
Evolution of a steel plate subject to a load from the UC San Diego blast simulator [16]

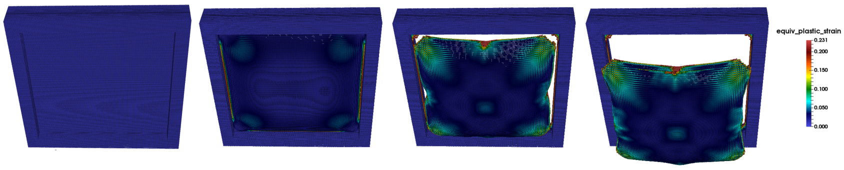
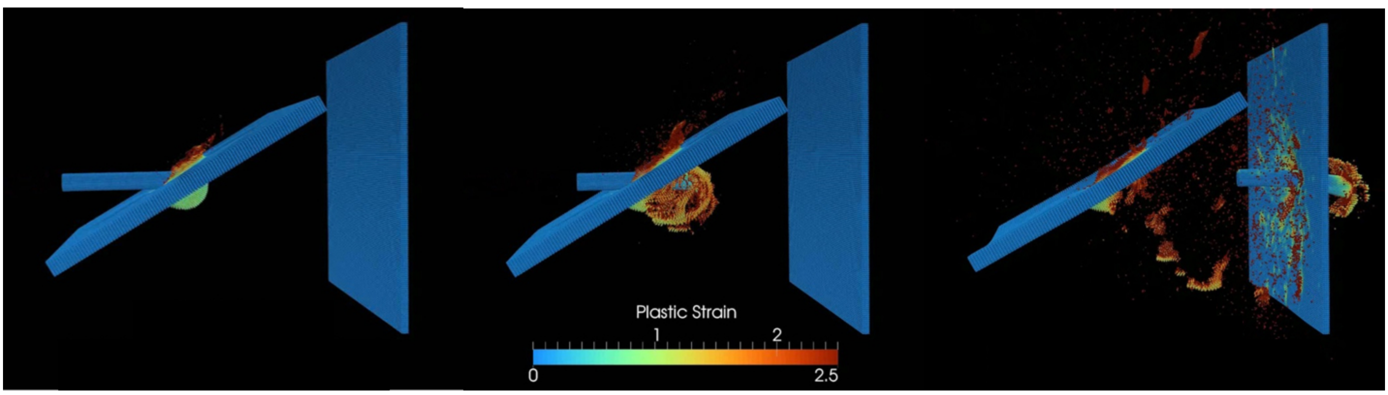


Figure 12:
Evolution of multi-layer impact problem with behind-armor debris field



(a) *(b)*

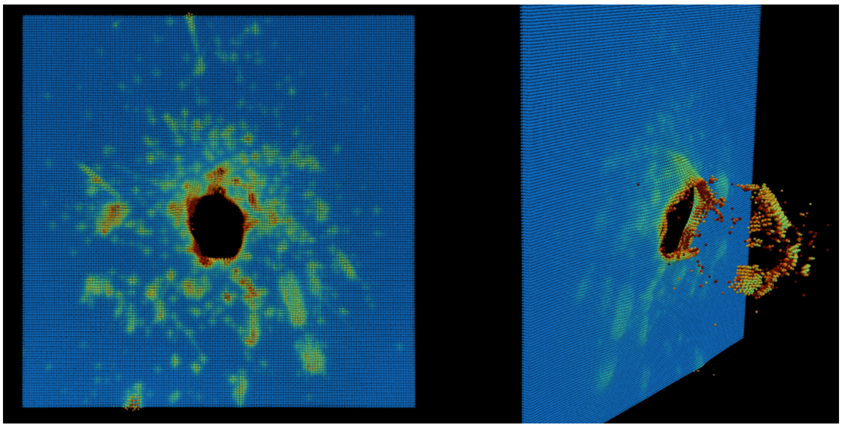


Figure 13:
Simulation of multi-layer impact with behind-armor debris field:
(a) front of witness plate: debris impact damage on second plate;
(b) back of witness plate: rod perforation of second plate and additional debris field.

of the reaction support frame. *Figure 12* and *Figure 13* show the simulation of a tungsten rod penetrating an oblique steel impact plate backed by a thin aluminum witness panel. A variety of penetration mechanisms and damage modes associated with ductile material failure are seen, where large plastic deformation, tearing, penetrator bending and debris field interaction are evident. In this class of problems, finite element techniques for penetration such as erosion would be ineffective at capturing the behind-armor debris; evaluating the safety of humans inhabiting a protective structure would not be possible. *Figure 14* shows the evolution of a landslide simulation with clear capturing of shear band formation, material flow, and shearing failure modes. ●

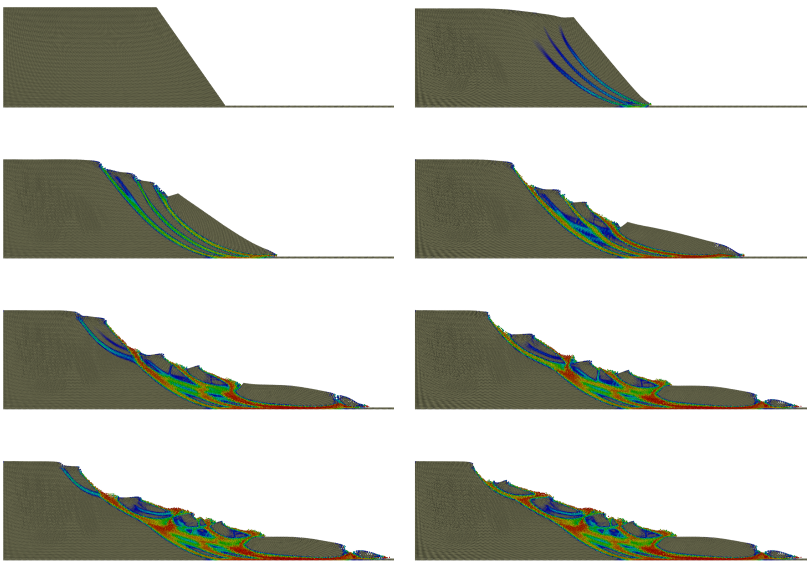


Figure 14:
Top left to bottom right: evolution of a landslide simulation using NSNI-RKPM.

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